Classical Compressed Sensing MRI Reconstruction with Sparse Priors and Undersampling Mask Robustness Analysis

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Background & Motivation

- Magnetic Resonance Imaging (MRI) acquires data in the Fourier (k-space) domain, requiring dense sampling to accurately reconstruct images, leading to long scan times
- Undersampling k-space can significantly accelerate the imaging, but introduces aliasing artificats in traditional reconstructions
- Compressed sensing addresses this by utilizing the sparsity of images in transform domains (wavelets or DCT) and using non-linear reconstruction techniques
- Iterative methods that combine data consistency in the Fourier domain with sparsity operations (such as soft-thresholding) enables recovery of high-quality images from incomplete measurements
- This work investigates alternative sparsity priors beyond wavelets, including DCT and total variation to improve MRI reconstruction performance

MRI Forward Model

$$y = M \cdot Fx + n$$

- ***x:*** underlying image
- ***F*:** Fourier transform (k-space)
- ***M*:** underlying mask
- ***y*:** measured k-space data
- ***n*:** measurement noise

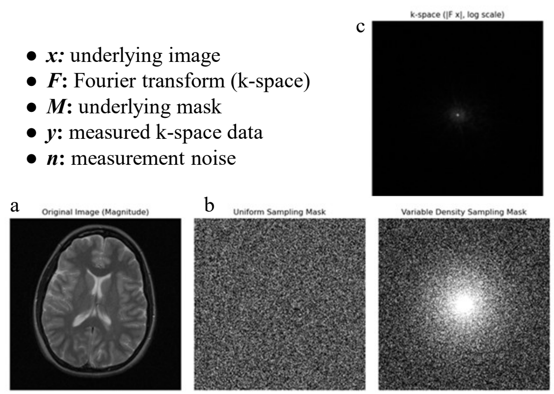

Figure 1. Forward model for MRI measurements. (a) Image x. (b) Sampling mask M, indicating acquired measurements. (c). Fourier transform of the image (k-space).

MRI measurement is modeled as sampling in the Fourier (k-space) domain. The image is transformed using the Fourier operator and then undersampled according to a sampling mask. This produces an undetermined inverse problem. Direct inversion produces aliasing artifacts due to missing frequency components. Accurate reconstruction, therefore, requires additional constraints such as sparsity.

Reconstruction With Alternative Sparsity Priors

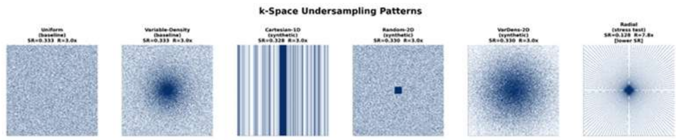

Figure 3. K-space undersampling masks used for robustness testing.

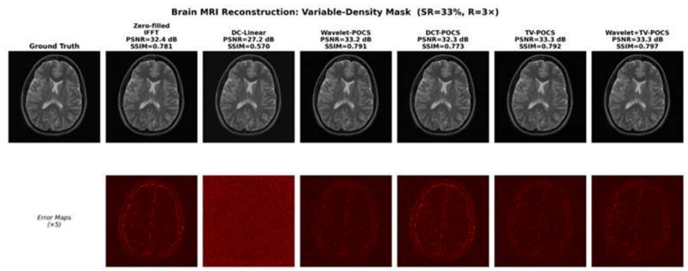

Figure 4. Reconstruction quality comparison across sparse priors under the variable-density mask. (20 iterations)

	Zero-filled IFFT	Wavelet-POCS	DCT-POCS	TV-POCS	Wavelet+TV POCS
Uniform	17.58	17.81	17.56	17.76	**17.90**
Var-Density	32.42	33.24	32.27	33.33	**33.34**
Cartesian-1D	29.11	29.86	29.02	30.63	**30.79**
Random-2D	19.76	20.37	19.72	20.06	**20.54**
VarDens-2D	30.41	32.53	30.51	31.38	**32.64**

Table 1. PSNR (dB) comparison across undersampling masks and reconstruction methods.

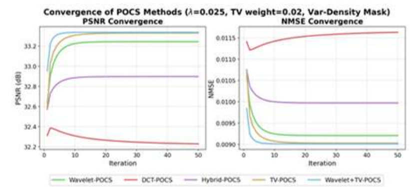

Figure 5. Convergence of POCS-based methods under the variable-density mask.

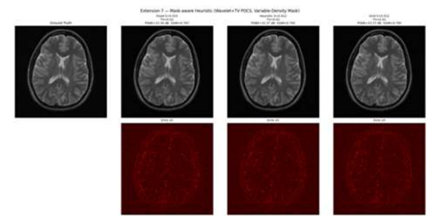

Figure 6. Representative reconstructions and error maps for Wavelet+TV methods.

Baseline Reconstruction

$$\hat{x} = \arg\min_{x} \frac{1}{2} \|M\mathcal{F}x - y\|_2^2 + \lambda \|x\|_1$$

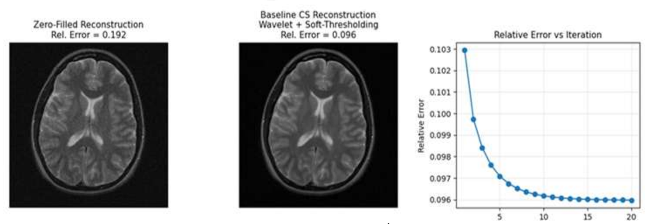

Figure 2. Baseline reconstruction. (a) Zero-filled reconstruction $F^{-1}(y)$, showing aliasing artifacts (b) Baseline reconstruction (using wavelet and soft-thresholding) which reduces artifacts (c). Relative error, demonstrating how reconstruction improves as algorithm runs.

Reconstruction Algorithm

1. Transform to wavelet domain
2. Apply soft-thresholding
3. Inverse Transform
4. Enforce data consistency in k-space
5. Repeat for multiple iterations

- **Data consistency term:** enforces agreement between the reconstructed image and the measured k-space data y at sampled locations defined by the mask M
- **$\lambda\|x_l\|$ (L1 regularization term):** promotes sparsity by encouraging many coefficients of x to be small or zero
- **λ:** controls the balance between these two terms

$\lambda = 0.025, TV_{weight} = 0.02, n_{iter} = 20$

Method	Best Mask	Sample Rate	PSNR	SSIM	NMSE	Runtime
Wavelet	Var-Den	33.3%	33.2425	0.7910	0.0092	1.9550 sec
DCT	Var-Den	33.3%	32.2652	0.7731	0.0115	1.7874 sec
TV	Var-Den	33.3%	33.3263	0.7924	0.0090	6.3667 sec
Wavelet+TV	Var-Den	33.3%	33.3367	0.7966	0.0090	8.6599 sec

Table 2. Best-mask quantitative comparison.

Conclusion

- CS-MRI reconstructs from undersampled k-space using Fourier consistency and sparsity priors, reducing aliasing vs. zero-fill
- Variable-density undersampling mask achieved the best reconstruction quality across all priors.
- Wavelet provides an efficient and strong baseline, while TV-POCS and Wavelet+TV-POCS further improve reconstruction quality at higher computational cost.

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